APPENDIX

A. Properties for Layering Cartoon Hair

This appendix details the last three layering properties in subsection 4.2. To begin, we first define the concept of *angles domain* for junction angles α and β. Since they locate at a common 2D junction, their sum should not exceed 2π. Moreover, without loss of generality, we assume α to be a larger angle, i.e., $\alpha \geq \beta$. Hence, the domain of α and β is basically the lower quarter of $[0, 2\pi] \times [0, 2\pi]$, which is the green area in Fig. 1(a).

Properties 2 to 4 supplement Property 1 for deciding the layering among cartoon hairs, see Fig. 1(b):

- Property 2 is based on the fact that when α and β have similar sizes (Fig. 2: example 1), we should not arbitrarily decide a layering between their related image regions solely by α and β, see Fig. 1(c).
- Property 3 accounts for a cartoon-hair-related situation, where multiple hair strands go below another hair strand altogether (Fig. 2: example 2). In this case, $\alpha+\beta \approx \pi$, and we again should not arbitrarily decide a layering between the two related regions solely by α and β, see Fig. 1(d).
- Lastly, Property 4 is related to another common situation with cartoon hairs (particularly due to image rasterization), where a hair tip lands on the edge of another hair strand (Fig. 2: example 3). Concerning this, when we compare α and β, we regard the region with a tiny sharp angle to be on the top because hair regions with sharp angles are very likely to be hair tips, i.e., β, which is the smaller angle, is close to zero. However, since we have to avoid the situations related to Properties 2 and 3, α should not be close to zero or π at the same time, see the blue areas in Fig. 1(e).

B. Algorithm: Hair Completion in Section 5 (Case 2)

Algorithm 1 below describes how we refine $\mathbf{C}_0$ by the combined vector field F_{ext} to form the completion area. Note that N is the normal of the curve; V_i is the velocity to push the curve outward; and $\mathbb{A}$ is a pentadiagonal matrix, see [30] for its detail.

Algorithm 1 ITERATIVE_REFINE $(\mathbf{C}_0, F_{ext})$

$i \Leftarrow 0$
while true do
  $i \Leftarrow i + 1$
  for each sample point $\mathbf{C}_{i-1}(t)$ **do**
    $V_i(t) \Leftarrow ||F_{ext}(\mathbf{C}_{i-1}(t)) \cdot N(\mathbf{C}_{i-1}(t))||N(\mathbf{C}_{i-1}(t))$
    $\mathbf{C}_i(t) \Leftarrow (I - \mathbb{A})^{-1}[\mathbf{C}_{i-1}(t) + V_i(t)]$
    if $\mathbf{C}_i(t)$ outside $O(R)$ **then**
      return $\mathbf{C}_{i-1}$
    end if
  end for
  if $||V_i|| < \epsilon$ **then**
    return $\mathbf{C}_i$
  end if
end while

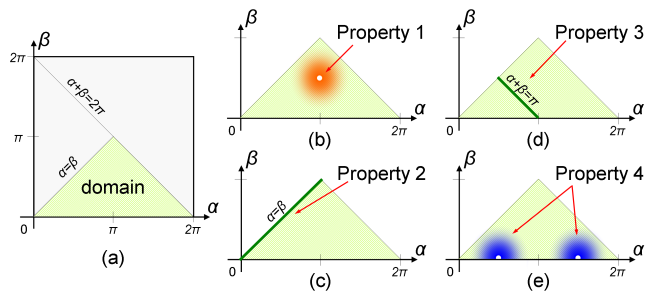

Fig. 1. (a) Domain of junction angles α and β; (b-e) Properties that define the junction metric for layering.

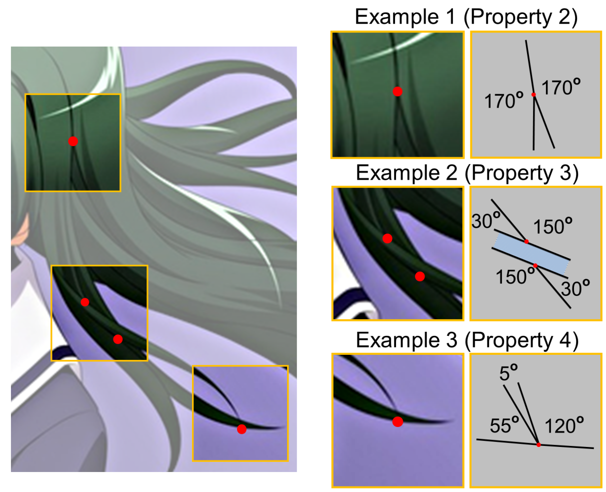

Fig. 2. Example Cartoon Hairs for Properties 2, 3 and 4.

C. Deformation Model for Local Hair Manipulation

The objective is defined as $\Omega = w_R\Omega_R + w_H\Omega_H + w_C\Omega_C$, where w_R, w_H and w_C are the weighting coefficients, which are set to $20, 1, 1000$, respectively in all our experiments. Ω_R is a deformation term defined as $\Omega_R = \sum_{i\in\mathbf{V},k\in\mathbf{S}} w_i^k \|(\mathbf{v}_i - \mathbf{s}_k) - \mathbf{T}_k(\mathbf{v}'_i - \mathbf{s}'_k)\|^2$, where w_i^k is the weight of vertex v_i for skeleton $s_k \in \mathbf{S}$ ($\mathbf{S}$ is a set of bones) and is set to be the inverse of the distance from the skeleton; and symbol $'$ indicates variables of the deformed skeleton. $\mathbf{T}_k$ is the transformation matrix to restrict the resizing of hair along the tangential direction of the skeleton to maintain the overall shape; it is defined as a rigid transformation:

$$\mathbf{T}_k = \mathbf{R}_{(1,0)}^{e_{k'}} \begin{bmatrix} s & 0 \\ 0 & 1 \end{bmatrix} \mathbf{R}_{e_k}^{(1,0)} ,$$

where $e_k = s_{k2} - s_{k1}$ and e'_k is the deformed edge vector, and $\mathbf{R}_v^{v'}$ is the rotation matrix that transforms vertex v to v'; and $s = \|e_{k'}\|/\|e_k\|$ is a scale factor. This transformation allows (and prevents) scaling along tangential (and normal) direction of the bones. Ω_H is a smoothness term defined as $\Omega_H = \sum_{(i,j)\in\mathbf{E}} w_{ij}\|\mathbf{v}'_j - \mathbf{v}'_i\|^2$, where $w_{ij} = cot\alpha_{ij} + cot\beta_{ij}$ are the discrete harmonic weights, α_{ij} and β_{ij} are the angles opposite to the edge (i, j) in the original mesh. Ω_C is a constraint term defined as $\Omega_C = \sum_{i\in\mathbf{C}} \|\mathbf{v}_i - \mathbf{v}'_i\|^2$, where $\mathbf{C}$ is a set of constraint vertices. We set boundary constraints at the root of the hair strand.